Letters

Comments on "On the Application of Complex Resistive Boundary Conditions to Model Transmission Lines Consisting of Very Thin Superconductors"

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In the above paper,[1] the authors present an elegant method to account for metallic losses in thin superconductors. The extensive numerical effort, which would otherwise be necessary to exactly solve the boundary-value problem, is considerably reduced by using the concept of complex resistive boundary conditions. This idea seems to have been originally introduced by Senior [1] as the authors point out.

In Section II, the authors introduce a sheet resistance R ((11))

$$R = \frac{1}{\sigma t} \tag{1}$$

where σ is the conductivity of the sheet and t its thickness. The assumption is then made that ((12))

$$\lim_{\substack{t \to 0 \\ \sigma \to \infty}} \frac{1}{\sigma t} = R. \tag{2}$$

What is the physical phenomenon responsible for the increase in the conductivity σ?

In Section III, the authors apply the method to analyze a lossy microstrip line through Galerkin's method. There seems to be, however, a crucial point that may invalidate their numerical results. Indeed, (44) and (45) contain **constant** diagonal elements in addition to the usual components of the lossless Green's impedance dyadics. The authors then expand the current density in a set of basis functions, each of which satisfies the **edge** condition ((46), (47)). Galerkin's method is then applied to determine the propagation properties of the structure.

The presence of the edge condition along with the constant term R in the diagonal elements of the Green's functions leads to integrals of the form

$$R \int_{-w/2}^{w/2} J_{0z}(y) J_{0z}(y)\, dy \propto \int_{-1}^{1} \frac{dy}{1 - y^2}. \tag{3}$$

which are infinite. Parseval's relation was used to calculate the above integral in ordinary space instead of the spectral domain. Also, only the lowest term J_{0z} was considered, but other ones are singular as well. Taking this observation into account, could the authors explain how they obtained numerical results that agree with the analytical solution to the parallel-plate transmission line?

In addition, and taking into account the divergency of the matrix elements, why isn't the attenuation constant infinite since it measures the losses in the structure which are given by (3):

$$\frac{1}{2} \int R J_s J_s^* \, ds \tag{4}$$

where J_s is the surface current? The integral in (4) is singular yet the authors present finite values for the attenuation constant (Fig. 9).

Manuscript received October 3, 1994

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IEEE Log Number 9408570.

[1] J. M. Pond, C. M. Krowne, and W. C. Carter, *IEEE Trans. Microwave Theory Tech.*, vol. 37, no. 1, pp. 181–190, Jan. 1989.

References

[1] T. B. A. Senior, "Half plane edge diffraction," *Radio Sci.*, vol. 10, pp. 645–650, June 1975.

[2] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics.* New York: Wiley, 1965, p. 290.

Reply to Comments on "On the Application of Complex Resistive Boundary Conditions to Model Transmission Lines Consisting of Very Thin Superconductors"

J. M. Pond and C. M. Krowne

The comments indicate a misunderstanding of several issues that are central to the above paper.[1] This response will address all three of those issues in the same sequence as they were raised. The first question concerns the resistive boundary condition. The remaining two issues concern the use of the spectral domain approach with the resistive boundary condition to calculate the propagation constant and loss of transmission line structures.

With respect to the first issue, there exists some confusion in the comments concerning an approximation, which is made to reduce the difficulty of the electromagnetic calculation, with a real physical process. By mathematically preserving the value of the sheet resistance, R, in the limit as the sheet thickness is mathematically reduced to zero, a three-region problem is reduced to a two-region problem with a modified boundary condition. As is pointed out in the above paper, the sheet resistance for a superconducting film which is thin compared to the superconducting penetration depth can be described by

$$\frac{1}{\sigma t} = R \tag{1}$$

where σ is the finite complex conductivity of the thin superconductor and t is the finite thickness thereof. In principle, the electromagnetic problem to be solved for the case of an infinite sheet is a three region problem, with region I comprising the region above the sheet, region II being the finite thickness sheet itself, and region III representing the region below the sheet. The electromagnetic

Manuscript received November 10, 1994.

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IEEE Log Number 9408571.

[1] J. M. Pond, C. M. Krowne, and W. C. Carter, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-37, no. 1, pp. 181–190, Jan. 1989.